Classical correlation and quantum discord sharing of Dirac fields in noninertial frame

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Abstract

The classical and quantum correlations sharing between modes of the Dirac fields in the noninertial frame are investigated. It is shown that: (i) The classical correlation for the Dirac fields decreases as the acceleration increases, which is different from the result of the scalar field that the classical correlation is independent of the acceleration; (ii) There is no simple dominating relation between the quantum correlation and entanglement for the Dirac fields, which is unlike the scalar case where the quantum correlation is always over and above the entanglement; (iii) As the acceleration increases, the correlations between modes I and II and between modes A and II increase, but the correlations between modes A and I decrease.

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I. INTRODUCTION

The integration of the quantum information and another fundamental part of modern physics theories—relativity theory gives birth to the theory of the relativistic quantum information [1–3]. It is believed that the investigation of the quantum correlation in a relativistic framework is not only helpful to understand some key questions in the quantum information theory, but also plays an important role in the study of the entropy and information paradox [4, 5] of the black hole. Following the pioneering work presented by Peres *et al.* [6], many authors [7–19] considered the quantum entanglement in a relativistic setting. Recently, Adesso *et al.* [20] discussed the entanglement sharing of a scalar field in the noninertial frame and found that the classical correlation is independent of the acceleration of the observer if the other observer stays stationary. Juan Len *et al.* studied the Dirac entanglement in both the spin and occupation number cases [21], and Mann *et al.* discussed the speeding up entanglement degradation of the scalar field in the noninertial frame [22]. Eduardo *et al.* investigated the the effect of the statistics on the entanglement [23] and showed that the entanglement survival of the Dirac field is fundamentally inherent in the Fermi-Dirac statistics and that it is independent of the number of the modes considered.

However, the entanglement is not the only characterization of a quantum system, and it was found that it has no advantage in some quantum information tasks. As seen in Refs. [24, 25], although there is no entanglement, certain quantum information processing tasks can also be done efficiently. Such a resource for the quantum computation and communication, i.e., the quantum correlation [26–28], is believed more practical than the entanglement. Moreover, it could be used to improve the efficiency of the quantum Carnot engine [29] and to get a better understand of the quantum phase transition [30]. More recently, Datta [31] calculated the quantum correlation between two relatively accelerated scalar modes, and showed that the quantum correlation which is measured by the quantum discord, is over and above the entanglement. In the limit of the infinite acceleration there is a finite amount of quantum correlation while the entanglement don't exist.

We noticed that most studies in the noninertial system focused on the entanglement, while the study of the classical correlation and how to distinguish the classical and quantum correlations is almost ignored. In fact, this is an important problem for mixed states since sometimes the quantum correlation is hidden by their classical correlation [30]. In this

paper we will discuss the sharing of the classical and quantum correlations of the Dirac fields in the noninertial frame, as well as a comparative study of the relationships between the entanglement and quantum correlation in this system. We are interested in how the acceleration will influence the classical and quantum correlations, and whether or not the differences between Fermi-Dirac and Bose-Einstein statistic will plays a role in the classical and quantum correlations sharing. We assume that two observers, Alice and Rob, share a entangled initial state at the same point in flat Minkowski spacetime. After the coincidence of them, Alice stays stationary while Rob moves with uniform acceleration a. It is well known that a uniformly accelerated observer in Rindler region I has no access to the field modes of the causally disconnected Rindler region II. Thus we must trace over the inaccessible modes, which leads the initial pure state to a mixed state. At the same time, from an inertial perspective the system is bipartite, but from a noninertial perspective an extra set of complementary modes in Rindler region II becomes relevant. Therefore, we have to calculate the classical and quantum correlations in all possible bipartite divisions of the tripartite system: the mode A described by Alice, the mode I in Rindler region I (described by Rob), and the complementary mode II in the Rindler region II.

The outline of the paper is as follows. In Sec. II we recall some concept from the view of the quantum information theory, in particular the classical and quantum correlations. In Sec. III we investigate the essential features of the Dirac fields in the noninertial frame. In Sec. IV we study the classical and quantum correlations sharing in this system. We summarize and discuss our conclusions in the last section.

II. CLASSICAL AND QUANTUM CORRELATIONS

We now present a brief review of the classical correlation and quantum discord. In classical information theory, the information can be quantified by Shannon entropy $H(X) = -\sum_x P_{|X=x} \log P_{|X=x}$, where $P_{|X=x}$ is the probability with X being x. For two random variables X and Y, the total correlation between them can be measured by the mutual information which is defined as $I(X:Y) = H(X) + H(Y) - H(X,Y)$, whose quantum version can be written as [32]

$$\mathcal{I}(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{1}$$

where $S(\rho) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy. By introducing the conditional entropy $H(Y|X) = H(Y, X) - H(X)$, we can rewrite the mutual information as

$$I(X : Y) = H(Y) - H(Y|X). \tag{2}$$

In order to generalize the above equation to the quantum domain, we measure the subsystem A of ρ_{AB} by a complete set of projectors $\{\Pi_j\}$, corresponds to the outcome j, which yields $\rho_{B|j} = Tr_A(\Pi_j \rho_{AB} \Pi_j)/p_j$, with $p_j = Tr_{AB}(\Pi_j \rho_{AB} \Pi_j)$. Then the quantum mutual information can alternatively defined by

$$\mathcal{J}_{\{\Pi_j\}}(A : B) = S(\rho_B) - S_{\{\Pi_j\}}(B|A), \tag{3}$$

where $S_{\{\Pi_j\}}(B|A) = \sum_j p_j S(\rho_{B|j})$ is conditional entropy [33] of the quantum state. The above quantity strongly depends on the choice of the measurements $\{\Pi_j\}$. In order to calculate the classical correlation, we shall minimize the conditional entropy over all possible measurements on A which corresponds to finding the measurement that disturbs least the overall quantum state and allows one to extract the most information about the state [26]. Then we define the classical correlation between parts A and B as

$$\mathcal{C}(A : B) = \max_{\{\Pi_j\}} \mathcal{J}_{\{\Pi_j\}}(A : B), \tag{4}$$

and the quantum discord [26] as

$$\mathcal{D}(A : B) = \mathcal{I}(A : B) - \mathcal{C}(A : B), \tag{5}$$

which presents even for separable states and arises as a consequence of coherence in a quantum system. In particular, a zero discord means the information of the quantum state can be obtained by observers without perturbing it, i.e, all the correlations are classical.

III. QUANTIZATION FOR DIRAC FILED IN MINKOWSKI SPACETIME AND ACCELERATED SYSTEM

For an inertial observer in flat Minkowski spacetime, the field can be quantized in a straightforward manner by expanding it in terms of a complete set of positive and negative frequency modes

$$\Psi = \int dk \, (a_{\mathbf{k}} \, \psi^+_{\mathbf{k}} + b^\dagger_{\mathbf{k}} \, \psi^-_{\mathbf{k}}), \tag{6}$$

where $\mathbf{k}$ is the wave vector which is used to label the modes and for massless Dirac fields $\omega = |\mathbf{k}|$. The above positive and negative frequency modes satisfy $\{a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k}'}\} = \{b_{\mathbf{k}}, b^{\dagger}_{\mathbf{k}'}\} = \delta(\mathbf{k} - \mathbf{k}')$ with all other anticommutators vanishing.

The appropriate coordinates to describe Rob's motion are the Rindler coordinates, given by

$$at = e^{a\varepsilon} \ \sinh(a\eta), \qquad az = e^{a\varepsilon} \ \cosh(a\eta). \tag{7}$$

The quantum field theory for the Rindler observer is constructed by expanding the field in terms of the complete set of positive and negative frequency modes [10]

$$\Psi = \int d\mathbf{k}[\hat{c}^{I}_{\mathbf{k}}\Psi^{I+}_{\mathbf{k}} + \hat{d}^{I\dagger}_{\mathbf{k}}\Psi^{I-}_{\mathbf{k}} + \hat{c}^{II}_{\mathbf{k}}\Psi^{II+}_{\mathbf{k}} + \hat{d}^{II\dagger}_{\mathbf{k}}\Psi^{II-}_{\mathbf{k}}], \tag{8}$$

where $\hat{c}^{I}_{\mathbf{k}}$ and $\hat{d}^{I\dagger}_{\mathbf{k}}$ are the fermion annihilation and antifermion creation operators acting on the state in region I, and $\hat{c}^{II}_{\mathbf{k}}$ and $\hat{d}^{II\dagger}_{\mathbf{k}}$ are the fermion annihilation and antifermion creation operators acting on the state in region II respectively. The canonical anticommutation relations of the mode operators are

$$\begin{aligned} \{\hat{c}^{I}_{\mathbf{k}}, \hat{c}^{I\dagger}_{\mathbf{k}'}\} &= \{\hat{d}^{I}_{\mathbf{k}}, \hat{d}^{I\dagger}_{\mathbf{k}'}\} = \delta(\mathbf{k} - \mathbf{k}'), \\ \{\hat{c}^{II}_{\mathbf{k}}, \hat{c}^{II\dagger}_{\mathbf{k}'}\} &= \{\hat{d}^{II}_{\mathbf{k}}, \hat{d}^{II\dagger}_{\mathbf{k}'}\} = \delta(\mathbf{k} - \mathbf{k}'), \end{aligned} \tag{9}$$

with all other anticommutators vanishing. All the above positive and negative frequency modes are defined using the future-directed timelike Killing vector: in inertial frame the Killing vector is ∂_t, in Rindler region I is ∂_η and in region II is $\partial_{-\eta}$.

We can easily get the Bogoliubov transformations [34] between the creation and annihilation operators of Rindler and Minkowski coordinates. After properly normalizing the state vector, the Minkowski vacuum is found to be an entangled two-mode squeezed state

$$|0_{\mathbf{k}}\rangle_M = \cos r|0_{\mathbf{k}}\rangle_I|0_{-\mathbf{k}}\rangle_{II} + \sin r|1_{\mathbf{k}}\rangle_I|1_{-\mathbf{k}}\rangle_{II}, \tag{10}$$

where $\cos r = (e^{-2\pi\omega/a}+1)^{-\frac{1}{2}}$ with a is Rob's acceleration. We noticed that in scalar case the Minkowski vacuum is $|0_{\mathbf{k}}\rangle_M = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n_{\mathbf{k}}\rangle_I|n_{-\mathbf{k}}\rangle_{II}$ with $\cosh r = (1-e^{-2\pi\omega/a})^{-\frac{1}{2}}$. The disparity between the scalar field and Dirac field is caused by the differences between Fermi-Dirac and Bose-Einstein statistic. Due to the Pauli exclusion principle, there are only two allowed states for each mode, $|0\rangle_M$ and $|1\rangle_M$ for fermions, and similarly for antifermions. Thus, the only excited state is given by

$$|1\rangle_M = |1_{\mathbf{k}}\rangle_I|0_{-\mathbf{k}}\rangle_{II}. \tag{11}$$

For simplicity we refer to the particle mode $\{|n_{\mathbf{k}}\rangle_I\}$ simply as $\{|n\rangle_I\}$, and the anti-particle mode $\{|n_{-\mathbf{k}}\rangle_{II}\}$ as $\{|n\rangle_{II}\}$. When Rob travels with uniform acceleration through the Minkowski vacuum, his detector registers the number of particles

$$N^2 = \frac{1}{e^{2\pi\omega/a}+1}, \tag{12}$$

which shows the accelerated observer detects a thermal Fermi-Dirac distribution of particles.

IV. CLASSICAL AND QUANTUM CORRELATIONS SHARING

The redistribution of entanglement in tripartite systems has been studied both in inertial [35] and noninertial systems [10, 20] by tracing over any one of the three qubits to calculate all the bipartite entanglement. Here we will use a similar method to discuss the quantum and classical correlation sharing in the accelerated fermi system. We assume that Alice has a detector which only detects mode $|n\rangle_A$ and Rob has a detector sensitive only to mode $|n\rangle_R$, they share a maximally entangled initial state

$$|\Phi\rangle_{AR} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_R + |1\rangle_A|1\rangle_R), \tag{13}$$

at the same point in flat Minkowski spacetime. After the coincidence of Alice and Rob, Alice stays stationary while Rob moves with uniform acceleration a. Using Eqs. (10) and (11), we can rewrite Eq. (13) in terms of Minkowski modes for Alice and Rindler modes for Rob

$$\begin{aligned}|\Phi\rangle_{A,I,II} &= \frac{1}{\sqrt{2}}(\cos r|0\rangle_A|0\rangle_I|0\rangle_{II} + \sin r|0\rangle_A|1\rangle_I|1\rangle_{II} \\ &\quad +|1\rangle_A|1\rangle_I|0\rangle_{II}).\end{aligned} \tag{14}$$

A. Physical accessible correlations

Since Rob is causally disconnected from the region II, the only information which is physically accessible to the observers is encoded in the mode A described by Alice and the mode I described by Rob. Taking the trace over the state of region II, we obtain

$$\rho_{A,I} = \frac{1}{2}\Big[\cos^2 r|00\rangle\langle 00| + \cos r(|00\rangle\langle 11| + |11\rangle\langle 00|) + \sin^2 r|01\rangle\langle 01| + |11\rangle\langle 11|\Big],$$

where $|mn\rangle = |m\rangle_A|n\rangle_I$. The von Neumann entropy of this state is $S(\rho_{A,I}) = -\frac{1+\cos^2 r}{2}\log_2(\frac{1+\cos^2 r}{2}) - \frac{1-\cos^2 r}{2}\log_2(\frac{1-\cos^2 r}{2})$. Similarly, we can obtain the entropy $S(\rho_A)$ for the reduced density matrix of the mode A and $S(\rho_{II})$ for the mode II, respectively.

Then let us make our measurements on the A subsystem, the projectors are defined as [24, 26]

$$\Pi_+ = \frac{I_2 + \mathbf{n}\cdot\sigma}{2} \otimes I_2, \qquad \Pi_- = \frac{I_2 - \mathbf{n}\cdot\sigma}{2} \otimes I_2, \tag{15}$$

where $n_1 = \sin\theta\cos\varphi,\ n_2 = \sin\theta\sin\varphi,\ n_3 = \cos\theta$ and σ_i are Pauli matrices. After the measurement of Π_+, the quantum state $\rho_{A,I}$ changes to

$$\begin{aligned}
\rho_{(I|+)} &= Tr_A(\Pi_+\rho_{A,I}\Pi_+)/p_+ = Tr_A(\Pi_+\rho_{A,I})/p_+ \\
&= \frac{1}{4p_+}Tr_A \begin{bmatrix} (1+\cos\theta)\cos^2 r & e^{i\varphi}\sin\theta\cos r & e^{-i\varphi}\sin\theta\cos^2 r & (1-\cos\theta)\cos r \\ 0 & (1+\cos\theta)\sin^2 r & 0 & e^{-i\varphi}\sin\theta\sin^2 r \\ 0 & 0 & 0 & 0 \\ (1+\cos\theta)\cos r & e^{i\varphi}\sin\theta & e^{-i\varphi}\sin\theta\cos r & 1-\cos\theta \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} (1+\cos\theta)\cos^2 r & e^{i\varphi}\cos r\sin\theta \\ e^{-i\varphi}\cos r\sin\theta & 1-\cos\theta+(1+\cos\theta)\sin^2 r \end{bmatrix},
\end{aligned} \tag{16}$$

where $p_+ = Tr(\Pi_+\rho_{A,I}\Pi_+) = 1/2$, $\Pi_+^2 = \Pi_+$, and $Tr(\alpha\beta) = Tr(\beta\alpha)$ were used. The eigenvalues of this density matrix are $\lambda_+(1,2) = \frac{1}{2}(1 \pm \sqrt{1 \pm \sin^2 2r\cos^4\frac{\theta}{2}})$. Similarly, we can obtain

$$\rho_{(I|-)} = \frac{1}{4p_-}\begin{bmatrix} (1-\cos\theta)\cos^2 r & -e^{i\varphi}\cos r\sin\theta \\ -e^{-i\varphi}\cos r\sin\theta & 1+\cos\theta+(1-\cos\theta)\sin^2 r \end{bmatrix}, \tag{17}$$

where $p_- = Tr(\Pi_-\rho_{A,I}\Pi_-) = 1/2$, which yields to $\lambda_-(1,2) = \frac{1}{2}(1 \pm \sqrt{1 \pm \sin^2 2r\sin^4\frac{\theta}{2}})$. Using Eqs. (16) and (17), we obtain the conditional entropy $S_{\{\Pi_j\}}(I|A) \equiv \sum_j p_j S(I|j)$. The classical correlation in this case is

$$\mathcal{C}(\rho_{A,I}) = -\frac{\cos^2 r}{2}\log_2(\frac{\cos^2 r}{2}) - (1-\frac{\cos^2 r}{2})\log_2(1-\frac{\cos^2 r}{2}) - \min_{\Pi_j} S_{\{\Pi_j\}}(I|A), \tag{18}$$

and the value of quantum discord can be given by

$$\begin{aligned}
\mathcal{D}(\rho_{A,I}) = 1 \ &+\ \frac{1+\cos^2 r}{2}\log_2(\frac{1+\cos^2 r}{2}) + \frac{1-\cos^2 r}{2}\log_2(\frac{1-\cos^2 r}{2}) \\
&+\ \min_{\Pi_j} S_{\{\Pi_j\}}(I|A).
\end{aligned} \tag{19}$$

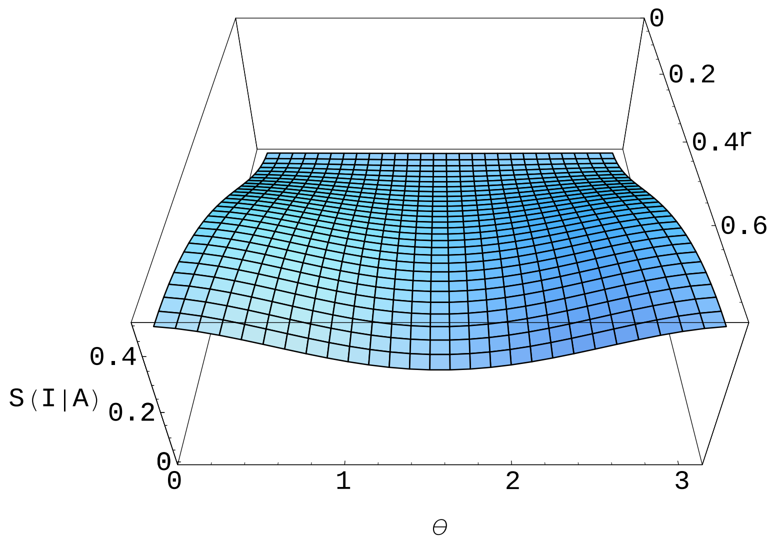

FIG. 1: (Color online) The condition entropy $S(I|A)$ as functions of r and θ.

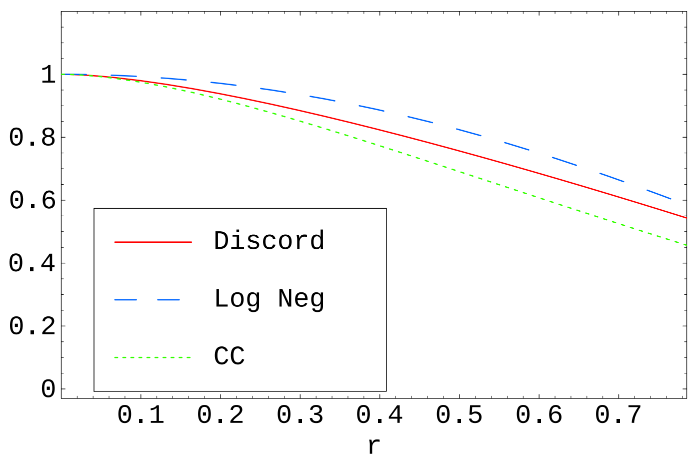

FIG. 2: (Color online) The discord(red line), logarithmic negativity (dashed blue line), and classical correlation (dotted green line) of $\rho_{A,I}$ as a function of r.

Note that the conditional entropy has to be numerically evaluated by optimizing over the angles θ and ϕ. Thus we should minimize it over all possible measurements on A, which corresponds to finding the measurement that disturbs least the overall quantum state. It is clear that the condition entropy is independent of φ, thus we plot it as functions of r and θ in Fig. 1, from which the minimum of the conditional entropy can be obtained when $\theta = \pi/2$. Then we can get the accurate value of Eqs. (18) and (19) and plot them in Fig. 2. For comparison, we also plot the logarithmic negativity [36] of the same state.

Fig. 2 shows how the acceleration changes the classical and quantum correlations. The monotonous decrease of $\mathcal{D}(\rho_{A,I})$ as the acceleration increases means that the quantum correlation of state $\rho_{A,I}$ decreases due to the thermal fields generated by the Unruh effect. Note that the classical correlation for the Dirac fields $\mathcal{C}(\rho_{A,I})$ also decreases as the acceleration increases, which is different from the result of scalar field that the classical correlation is independent of the acceleration [20]. It is interesting to note that in the Dirac case entanglement of $\rho_{A,I}$ is always larger than quantum correlation, which is in sharply contrast to the result of the scalar fields that the quantum correlation is always over and above entanglement [31]. This obvious distinction is caused by the differences between Fermi-Dirac and Bose-Einstein statistics. The Dirac particles which have half-integer spin must obey the Pauli exclusion principle and access to only two quantum levels.

B. Physical unaccessible correlations

To explore correlations in this system in detail we consider the tripartite system consisting of the modes A, I, and II. We therefore calculate the correlations in all possible bipartite divisions of the system. Let us first comment on the correlations created between the mode A and mode II, tracing over mode I of the state Eq. (14) , we obtain the density matrix

$$\rho_{A,II}=\frac{1}{2}\bigg[\cos r^2|00\rangle\langle 00|+\sin r(|10\rangle\langle 01|+|01\rangle\langle 10|)+\sin^2 r|01\rangle\langle 01|+|10\rangle\langle 10|\bigg],$$

where $|mn\rangle = |m\rangle_A|n\rangle_{II}$. We can easily get $S(\rho_{A,II}) = -\frac{1}{2}\cos^2 r\log_2\frac{\cos^2 r}{2} - (1-\frac{1}{2}\cos^2 r)\log_2(1-\frac{1}{2}\cos^2 r)$. After these measurements, the state $\rho_{A,II}$ changes to

$$\frac{1}{4p'_\pm}\begin{bmatrix} 1\mp\cos\theta+(1\pm\cos\theta)\cos^2 r & \pm e^{i\varphi}\sin r\sin\theta \\ \pm e^{-i\varphi}\sin r\sin\theta & (1\pm\cos\theta)\sin^2 r \end{bmatrix}, \tag{20}$$

where $p'_+=p'_-=1/2$. According to the preceding calculations, we have

$$\mathcal{C}(\rho_{A,II})=-\frac{1+\cos^2 r}{2}\log_2(\frac{1+\cos^2 r}{2})-\frac{1-\cos^2 r}{2}\log_2(\frac{1-\cos^2 r}{2})-\min_{\Pi_j}S_{\{\Pi_j\}}(II|A) \tag{21}$$

and

$$\begin{aligned}\mathcal{D}(\rho_{A,II})=1\ &+\ \frac{1}{2}\cos^2 r\log_2\frac{\cos^2 r}{2}+(1-\frac{1}{2}\cos^2 r)\log_2(1-\frac{1}{2}\cos^2 r)\\ &+\ \min_{\Pi_j}S_{\{\Pi_j\}}(II|A).\end{aligned} \tag{22}$$

Similarly, the minimum of the conditional entropy $S_{\{\Pi_j\}}(II|A) \equiv \sum_j p_j S(II|i)$ can be obtained when $\theta = \pi/2$. Tracing over the modes in A, we obtain the density matrix

$$\rho_{I,II} = \frac{1}{2}\Big[\cos r^2|00\rangle\langle 00| + \sin r\cos r(|00\rangle\langle 11| + |11\rangle\langle 00|) + |10\rangle\langle 10| + \sin^2 r|11\rangle\langle 11|\Big],$$

where $|mn\rangle = |m\rangle_I|n\rangle_{II}$. The von Neumann entropy of this matrix is $S(\rho_j) = -\frac{1}{2}\cos^2 r\log_2\frac{\cos^2 r}{2} - (1-\frac{1}{2}\cos^2 r)\log_2(1-\frac{1}{2}\cos^2 r)$. After those measurements, the state $\rho_{I,II}$ changes to

$$\frac{1}{4p''_\pm}\begin{bmatrix} 1\mp\cos\theta + (1\pm\cos\theta)\cos^2 r & \pm e^{i\varphi}\sin r\cos r\sin\theta \\ \pm e^{-i\varphi}\sin r\cos r\sin\theta & (1\mp\cos\theta)\sin^2 r \end{bmatrix}, \tag{23}$$

where $p''_\pm = \frac{1}{2}(1\mp\cos\theta\sin^2 r)$. The classical correlation of $\rho_{I,II}$ is

$$\mathcal{C}(\rho_{I,II}) = -\frac{1+\cos^2 r}{2}\log_2(\frac{1+\cos^2 r}{2}) - \frac{1-\cos^2 r}{2}\log_2(\frac{1-\cos^2 r}{2}) - \min_{\Pi_j} S_{\{\Pi_j\}}(II|I), \tag{24}$$

and the quantum correlation is

$$\begin{aligned}\mathcal{D}(\rho_{I,II}) = &- \frac{1}{2}\cos^2 r\log_2\frac{\cos^2 r}{2} - (1-\frac{1}{2}\cos^2 r)\log_2(1-\frac{1}{2}\cos^2 r) - 1 \\ &+ \min_{\Pi_j} S_{\{\Pi_j\}}(II|I). \end{aligned}\tag{25}$$

The only difference is that we can get the minimum of $S_{\{\Pi_j\}}(II|I) \equiv \sum_j p_j S(II|j)$ when $\theta = \pi/4$.

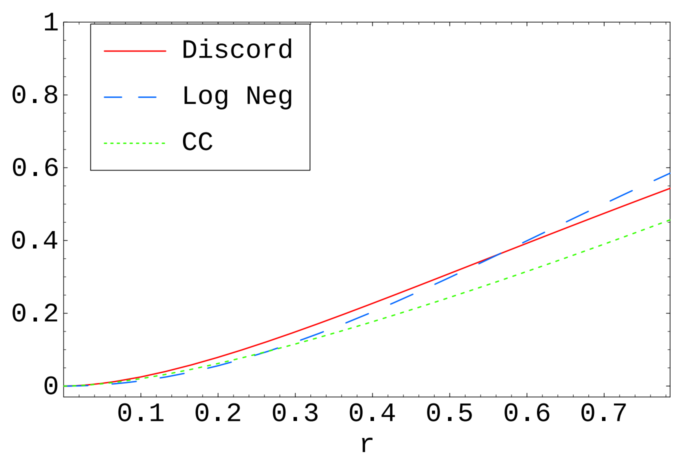

FIG. 3: (Color online) The discord(red line), logarithmic negativity(dashed blue line), and classical correlation(dotted green line) of $\rho_{A,II}$ as a function of r.

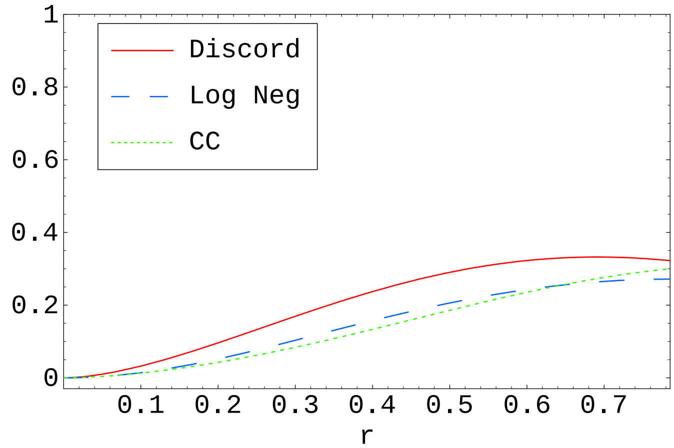

FIG. 4: (Color online) The discord(red line), logarithmic negativity(dashed blue line), and classical correlation(dotted green line) of $\rho_{I,II}$ as a function of r.

The properties of the correlations of $\rho_{A,II}$ and $\rho_{I,II}$ are shown in Figs.(3) and (4). They demonstrate that both the classical and quantum correlations of these two states increase as the acceleration increases. It is interesting to note that, for the state $\rho_{A,II}$, the quantum correlation dominates the entanglement when r is small, and the dominance is reversed as the acceleration increases. While for the state $\rho_{I,II}$, the quantum correlation always dominates the entanglement. That is to say, the quantum correlation can be larger than the entanglement for some states, whereas it can be smaller for other states. Thus, we arrive at the conclusion that there is no simple dominating relation between the quantum correlation and the entanglement in the noninertial frame.

In Figs. (5) and (6) we plot the redistributions of the quantum and classical correlation which show how the acceleration changes all the bipartite correlations. For lower acceleration, modes A and I remain almost maximally correlated while there is little correlations between modes I and II and between modes A and II. As the acceleration grows, the unaccessible correlations between modes I and II and between modes A and II increase, while the accessible correlation between modes A and I decreases. Consequently, the original two mode correlations in the state Eq. (13) described by Alice and Rob from an inertial perspective, are redistributed among the mode A described by Alice, the mode I described by Rob, and the complementary mode II. That is to say, the initial correlations described by inertial observers are redistributed between all the bipartite modes. Therefore, as a consequence of

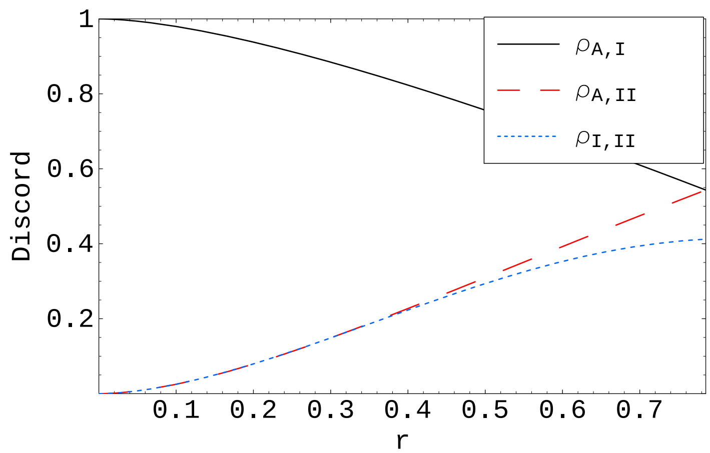

FIG. 5: (Color online) The quantum discords $D(\rho_{a,b})$ of $\rho_{A,I}$(black line), $\rho_{A,II}$(dashed red line) and $\rho_{I,II}$(dotted blue line) as a function of r. In the limit of infinite acceleration, i.e. $r \to \pi/4$, the discord of $\rho_{A,I}$ equals to the discord of $\rho_{A,II}$.

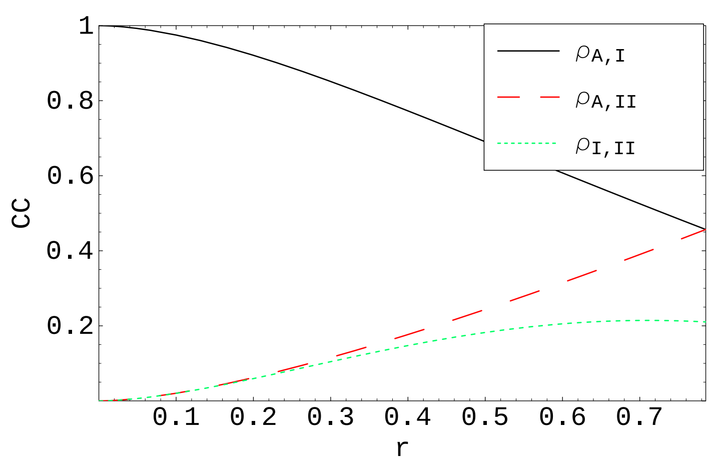

FIG. 6: (Color online) The classical correlations of $\rho_{A,I}$(black line), $\rho_{A,II}$(dashed red line) and $\rho_{I,II}$ (dotted green line) as a function of r. In the case of $r \to \pi/4$, the $\mathcal{C}(\rho_{A,I})$ also equals to $\mathcal{C}(\rho_{A,II})$.

the monogamy of correlations, the physically accessible correlations between the two modes described by Alice and Rob degrade. In the limit of infinite acceleration, the correlations of $\rho_{A,I}$ equal to the correlations of $\rho_{A,II}$. It is worth to mention that the authors in Ref. [31] only discussed the quantum correlation between Alice and the Rindler region I, while we discussed the redistribution of quantum correlation and classical correlation among all, accessible and unaccessible modes. The analysis of the unaccessible correlations between

Alice and region II, as well as regions I and II were used to explain the loss of accessible correlations between Alice and region I. The loss of correlations in accessible modes were redistributed to the unaccessible modes.

V. SUMMARY

The effect of the acceleration on the redistribution of the classical and quantum correlations of the Dirac fields in the noninertial frame is investigated. It is shown that: (i) The classical correlation $\mathcal{C}(\rho_{A,I})$ for the Dirac fields decreases as the acceleration increases, which is different from the result of the scalar case that the classical correlation is independent of the acceleration [20]. (ii) In the Dirac case, we find that the entanglement always dominates the quantum correlation for the state $\rho_{A,I}$, the dominating relation of the entanglement and quantum correlation is reversed at a point as the acceleration increases for the state $\rho_{A,II}$, while the quantum correlation always dominates the entanglement for the state $\rho_{I,II}$. These results are in sharply contrast to the result of the scalar case that the quantum correlation is always over and above the entanglement [31]. Thus, there is no simple dominating relation between the quantum correlation and the entanglement for the Dirac fields in the noninertial system. And (iii) as the acceleration increases, the correlations between modes I and II and between modes A and II increase, while the correlations between modes A and I decrease. Thus, the original correlations described by Alice and Rob from an inertial perspective are redistributed between all the bipartite modes from a noninertial perspective.

If we consider Alice to be accelerated as well, the increase and decrease of the correlations would be more obvious as the acceleration increases. The results of this paper can be also applied to the case that Alice stays stationary at the asymptotically flat region of a black hole while Rob barely escapes through it with eternal uniform acceleration. Such topics are left for a future research.

Acknowledgments

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